§ 9.1 Rational Numbers | HW: 8, 12, 16, 18, 20, 24bdfh, 26, 30, 38, 46, 48

Defn: A rational number is a number that can be written as $\frac{a}{b}$ where a, b are integers and $b \neq 0$.

e.g. $\frac{1}{2}, \frac{-5}{3}, \frac{73}{45}, \frac{6}{-12}, 0, \frac{3}{1}, \ldots$

The set of rational numbers is denoted $\mathbb{Q} = \{\frac{a}{b} \mid a, b \in \mathbb{Z} \text{ and } b \neq 0\}$.

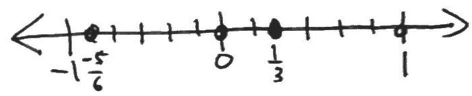
- Two rational numbers $\frac{a}{b}, \frac{c}{d}$ are equal if and only if $ad = bc$ (Cross-Multiplication).

e.g. $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$ since $4 \cdot 1 = 2 \cdot 2$ and $2 \cdot 6 = 3 \cdot 4 = 12$.

$\frac{-3}{7} \neq \frac{-5}{9}$ since $-3 \cdot 9 = -27 \neq -35 = -5 \cdot 7$.

We can graph rational numbers on a number line as we did with fractions & integers.

ex: $\frac{1}{3}, -\frac{5}{6}, 1, 0$:



We have a relationship between natural numbers ($\mathbb{N}$), whole numbers, integers ($\mathbb{Z}$), fractions, and rationals:

natural numbers
↓
whole numbers
↓ ↘
integers   fractions = positive terminating or repeating decimals
↘ ↓
rational numbers = terminating or repeating decimals (+ or −)

- Recall for a fraction we need a non-negative number. Rationals can be positive or negative or 0.

Notation: By definition, we can write $\frac{-1}{4}$ or $\frac{1}{-4}$ but the usual way to write this is $-\frac{1}{4}$, with the leading negative.

"$\mathbb{Z}$ is a subset of $\mathbb{Q}$"

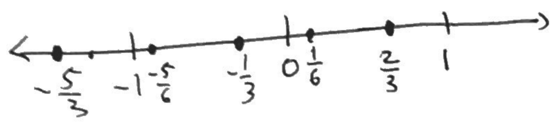
Ex: Justify "every integer is a rational number" i.e., $\mathbb{Z} \subseteq \mathbb{Q}$.

Proof: For any integer a, $a = \frac{a}{1}$, so $a \in \mathbb{Q}$.

Comparing Rational Numbers

- We compare rational numbers using (sometimes implicitly) a number line.
- $\frac{a}{b} < \frac{c}{d}$ if and only $\frac{a}{b}$ is left of $\frac{c}{d}$ on a number line

e.g: Order $-\frac{5}{3}, \frac{2}{3}, -\frac{1}{3}, -\frac{5}{6}, \frac{1}{6}$ from least to greatest



So, $-\frac{5}{3} < -\frac{5}{6} < -\frac{1}{3} < \frac{1}{6} < \frac{2}{3}$

Why Rational Numbers?

We started talking about numbers with the natural numbers $\{1, 2, 3, 4, \dots\}$ and gradually "filled in holes" in the number line to get whole numbers, ~~fr~~fractions, integers (for 0, parts of a whole, ~~neg~~ amounts KO resp). Now, we use rationals to get all quotients of integers.

Properly, we need some terminology:

Defn: A set (of numbers) is closed under an operation if the result of applying the operation to two elements of our set is still in the set.

- Our set $\mathbb{Q}$ of rational numbers is closed under the operations $+, -, \cdot, \div$, (addition, subtraction, multiplication, division)
- Each of our previous sets was not closed under at least one of the operations and was closed under at least one.
- Ex?

- Time for Worksheet (in pairs)

Operations on Rational Numbers

The ~~old~~ rules for $+/-/\cdot/\div$ with $\mathbb{Q}$ are exactly the same as the rules for fractions combined with the rules for integers that you saw last semester.

$\boxed{\text{Ex}}$ a) $-\frac{4}{5} + \frac{11}{5} = \frac{-4+11}{5} = \frac{7}{5} = 1\frac{2}{5}$

b) $-\frac{4}{5} - \frac{1}{2} = -\frac{8}{10} - \frac{5}{10} = \frac{-8-5}{10} = \frac{-13}{10} = -1\frac{3}{10}$

c) $-\frac{4}{5} \cdot \frac{3}{2} = \frac{-4 \cdot 3}{5 \cdot 2} = -\frac{12}{10} = \frac{-6}{5} = -1\frac{1}{5}$

d) $-\frac{4}{5} \div \frac{1}{5} = -\frac{4}{5} \cdot \frac{5}{1} = \frac{-4 \cdot 5}{5 \cdot 1} = \frac{-4}{1} = -4$